**B.Sc. (Honours) Examination, 2019**
**Semester-III (CBCS)**
**Statistics**
**Course : CC-7**
**(Mathematical Analysis)**

**Time : 3 Hours** **Full Marks : 60**

Questions are of value as indicated in the margin

*Group – A (Answer any ten questions)* $10 \times 1 = 10$

1. Answer the following questions with proper justification.

(a) State 'well ordering principle' and 'density property' of a set.

(b) How do you represent $\sqrt{7}$ on directed line?

(c) Write down the definition of a Borel set and give an example.

(d) Discuss the statement: $\{|x_n|\}$ is a convergent sequence that converges to $|l|$ then $\{x_n\}$ is a convergent sequence that converges to $l$.

(e) A convergent sequence is a Cauchy sequence. Does the converse of this statement is true?

(f) Discuss the statement: If $\sum u_n$ be a convergent series of positive real numbers then the series $\sum \sin^2 u_n$ is also convergent.

(g) The series $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$ is convergent?

(h) Find $\lim_{x\to 0} \cos\left(\frac{1}{x}\right)$.

(i) What do you mean by removable discontinuity? Give an example and discuss.

(j) Let $f : \boldsymbol{R} \to \boldsymbol{R}$ be a differentiable and $f'$ is continuous. Then find $\lim_{n\to\infty} (n+1)\int_0^1 x^n f(x)dx$.

(k) Let $f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$, $x \in \boldsymbol{R}$. Then find the exact number of distinct real roots of the equation $f'(x) = 0$.

(l) Which kind of interpolation formula we use for interpolating near the end of the tabular values with unequal space?

(m) What is the relation between $\nabla$ and $E$ operator?

(n) Write down the Stirlings approximation of $n!$.

*Group – B (Answer any five questions)* $5 \times 6 = 30$

2. (a) Show that $\sqrt[3]{6}$ is not a rational number.

(b) Show that the finite union and intersection of two neighbourhood of $\alpha$ is also a neighbourhood of $\alpha$. Does these results hold for infinite case? 3+3

P.T.O.

3. (a) If $\{x_n\}$ converges to $l$ then show that $\{e^{x_n}\}$ converges to $e^l$.
(b) Show that $\lim_{n\to\infty} \frac{n!}{n^n} = 0$.
(c) Find $\lim_{n\to\infty} \left[\left(\frac{2}{1}\right)\left(\frac{3}{2}\right)^2 \cdots \left(\frac{n+1}{n}\right)^n\right]^{1/n}$. 2+2+2

4. (a) For the bounded sequences $\{x_n\}$, $\{y_n\}$, show that $\overline{\lim}\,(x_n + y_n) \leq \overline{\lim}\, x_n + \overline{\lim}\, y_n$.
(b) Show that the sequence $\{x_n\}$ where $x_n = 1-\frac{1}{2}+\frac{1}{3}-\cdots+(-1)^{n-1}\frac{1}{n}$ is a Cauchy sequence. Hence discuss its convergence. 3+3

5. (a) State and prove Pringsheim theorem. Does the converse of this theorem is true?
(b) Test the convergence of the series $\sum[1-\cos(\pi/n)]$. 4+2

6. (a) Let $\{a_n\}$ be a sequence of real numbers such that $\sum a_n$ converges absolutely. Prove that the series $\sum \ln(1+a_n^4)$ converges.
(b) Discuss the convergence of: $a+b+a^2+b^2+a^3+b^3+\cdots$, where $0 < a < b < 1$. 3+3

7. (a) Show that $\lim_{x\to+\infty} \frac{[x]}{x} = 1$.
(b) Cite some examples where: *i*) Sum of two discontinuous function is continuous, *ii*) Product of two discontinuous function is continuous.
(c) For $x \in \boldsymbol{R}$, define $f(x) = \cos(\pi x) + [x^2]$ and $g(x) = \sin(\pi x)$. Which of the following statements is (are) true? (i) $f(x)$ is continuous at $x = 2$. (ii) $g(x)$ is continuous at $x = 2$. (iii) $f(x)+g(x)$ is continuous at $x = 2$. (iv) $f(x)g(x)$ is continuous at $x = 2$. 2+2+2

8. (a) Let $f : [0, 13] \to \boldsymbol{R}$ be defined by $f(x) = x^{13} - e^{-x} + 5x + 6$. Then find the minimum value of the function $f$ on $[0, 10]$.
(b) If $f, g, h$ are continuous on $[a, b]$ and differentiable on $(a, b)$, then show that $\exists c \in (a, b)$ such that $\begin{vmatrix} f(a) & g(a) & h(a) \\ f(b) & g(b) & h(b) \\ f'(c) & g'(c) & h'(c) \end{vmatrix} = 0$.
(c) Let $f : \boldsymbol{R} \to \boldsymbol{R}$ be differentiable function with $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} f'(x) = 2$. Then find the value of $\lim_{x\to\infty}\left(1 + \frac{f(x)}{x^2}\right)^x$. 2+2+2

9. (a) Solve the difference equation: $\nabla^4 x(n) = 0$.
(b) Derive the expression of error in Simpson's $1/3^{rd}$ Rule. 3+3

<u>*Group – C (Answer any two questions)*</u> $2 \times 10 = 20$

10. (a) Apply Mean Value Theorem prove that if $\phi(x) = F(f(x))$, then $\phi'(x) = f'(x)F'(f(x))$. Here the derivatives are continuous.
(b) Find $\lim_{x\to0+}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$.
(c) Show that $x - \frac{x^3}{3!} < \sin(x) < x - \frac{x^3}{3!} + \frac{x^5}{5!}$, for $x > 0$. 4+3+3

11. (a) State and prove Gauss's Forward Interpolation Formula.
(b) Using Lagrange's formula express $g(x) = \frac{x^2+x-3}{x^3-2x^2-x+2}$ as a sum of partial fractions.
(c) Find the missing value in the following table: 4+2+4

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | : | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| $f(x)$ | : | 1 | 3 | – | 73 | 225 | – | 1153 |

12. (a) Let $a_1 = 2,\ b_1 = 1$ and for $n \geq 1,\ a_{n+1} = \frac{a_n+b_n}{2},\ b_{n+1} = \frac{2a_nb_n}{a_n+b_n}$. Then show that *i)* $b_n \leq a_n,\ \forall n$, *ii)* $b_{n+1} \geq b_n,\ \forall n$, *iii)* $\{a_n\},\ \{b_n\}$ converges to the same limit $\sqrt{2}$.
(b) Discuss the convergence of the series: $1 + \frac{\alpha^2}{1.\beta} + \frac{\alpha^2(\alpha+1)^2}{1.2.\beta(\beta+1)} + \frac{\alpha^2(\alpha+1)^2(\alpha+2)^2}{1.2.3.\beta(\beta+1)(\beta+2)} + \cdots$.
(c) Discuss the convergence of the series: $\sum \frac{(-1)^{n-1}}{n\sqrt{n}}$ 4+4+2